A model of flavors

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We argue in favor of dynamical mass generation in an $SU(2)_L \times U(1)_Y$ electroweak model with two complex scalar doublets with ordinary masses. The masses of leptons and quarks are generated by ultraviolet-finite non-perturbative solutions of the Schwinger–Dyson equations for full fermion propagators with self-consistently modified scalar boson exchanges. The W and Z boson masses are expressed in terms of spontaneously generated fermion proper self-energies in the form of sum rules. The model predicts two charged and four real neutral heavy scalars.

The standard model of electroweak interactions is the best what in theoretical particle physics we have: In an operationally well defined framework it parameterizes and successfully correlates virtually all electroweak phenomena. Yet as an effective field theory it is ill with unnaturalness [1]: Due to the necessity of its quadratic renormalization there is no natural way how the Higgs mass could be made reasonably small. Natural ways end up with the Higgs mass uninterestingly high, of the order of the cutoff $\Lambda \approx 10^{16}$ GeV.

Possibilities of curing unnaturalness we are aware of are not numerous. First, supersymmetry models avoid the quadratic divergences associated with scalar fields by extending the field spectrum to become fermion-boson symmetric. Second, the 'Little Higgs' models (see [2] for introduction) avoid quadratic divergences associated with the Higgs boson mass at low energies by introducing new symmetries which generate sophisticated mixings between bosons. Third, the models without elementary scalar fields [3, 4] avoid the quadratic divergences by assumption. Unlike the first two possibilities they are necessarily strongly coupled and, in fact, they are not operationally well defined.

With the ultimate aim of generating the particle masses softly and with not vastly different couplings we suggest in this note to add to the list of models of spontaneous mass generation a dynamical one within an effective field theory description of the electroweak phenomena: Massive scalar fields distinguish fermions at tree level by their different $SU(2)_L \times U(1)_Y$ invariant Yukawa couplings, while spontaneous breakdown of this symmetry i.e., emergence of both the fermion and the intermediate-boson masses is a genuinely quantum self-consistent loop effect. This alternative is arguably also natural.

In a self-explanatory notation our $SU(2)_L \times U(1)_Y$ gauge-invariant electroweak model is defined by its Lagrangian $\mathcal{L}$ which consists of (i) Four standard massless gauge fields; (ii) n_f standard massless fermion families extended by n_f neutrino right-handed singlets with zero weak hypercharge together with their general Majorana mass matrix M_M; (iii) two distinct complex scalar doublets $S = (S^{(+)}, S^{(0)})$ and $N = (N^{(0)}, N^{(-)})$ with weak hypercharges $Y(S) = +1$ and $Y(N) = -1$, respectively, with different *ordinary* masses M_S and M_N, respectively, and with different respective self-couplings λ_S, λ_N. With spontaneous breakdown of the underlying symmetry in the scalar sector at tree level such a Lagrangian would define a popular extension of the Standard model [5].

The scalar field doublets are of utmost importance for the 'low-energy' physics of electroweak symmetry breaking. Their Yukawa couplings, which we assume to have the form

$$\mathcal{L}_Y = \bar{l}_L y_e e_R S + \bar{l}_L y_\nu \nu_R N + \\ + \bar{q}_L y_d d_R S + \bar{q}_L y_u u_R N + \text{H.c.}, \quad (1)$$

distinguish between otherwise identical fermion families, and break down explicitly all unwanted and dangerous inter-family symmetries. Both M_M and S, N together with their Yukawa couplings are thought of as remnants of an unknown high-energy dynamics. Because scalars ought to remain in the physical spectrum, and the neutral ones mediate in general the flavor-changing electric charge conserving transitions, for safety reasons we restrict their masses as $M_S, M_N \gtrsim 10^6$ GeV.

The troublesome quadratic renormalizations of M_S, M_N can easily be avoided at will by imposing mass-independent relations between the gauge couplings g, g', the Yukawa couplings y, and the scalar self couplings λ [6]. The Yukawa couplings y_e, y_ν, y_d, y_u are in general arbitrary complex constant $n_f \times n_f$ matrices unconstrained by the symmetry. The Lagrangian $\mathcal{L}$ is at least one-loop renormalizable off mass shell by power counting, and all its counter terms are only logarithmically divergent. At very high momenta at which all dynamically generated masses can be neglected it is possible to diagonalize two general complex matrices, say y_ν and y_u, by biunitary transformations into the real non-negative form without changing physics.

It should, however, be the dynamical issue of small momenta, what are the observable masses of leptons, quarks, and intermediate vector bosons.

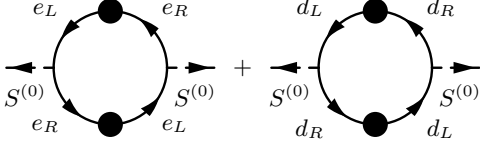I. Our intention is to break down the sacred $SU(2)_L \times$

FIG. 1: One-loop graphs that induce mixing of the real and imaginary components of the neutral scalar fields, leading to their mass splitting. The solid circles denote the chirality-changing part of the fermion self-energy $\Sigma(p)$. The same graphs also apply to $N^{(0)}$ upon replacing e, d with ν, u.

FIG. 2: One-loop amplitude for the charged scalar mixing induced by the Yukawa interaction.

$U(1)_Y$ symmetry dynamically. This amounts to first *assuming*, and then self-consistently *finding* non-zero fermion proper self-energies $\Sigma(p)$ due to the interactions. Their chirality-changing part must come out necessarily ultraviolet-finite because the fermion mass counter terms are strictly forbidden by the underlying chiral $SU(2)_L \times U(1)_Y$ symmetry. Like in the Standard model we assume also here that the fermion–gauge boson interactions are not the cause of the fermion masses, and they will therefore not be considered in this respect. The assumption is of course natural because these interactions do not feel flavors. Like in the Standard model the fermion masses will be generated by the Yukawa couplings of fermions with scalar doublets which do feel flavors. Unlike the Standard model the fermion masses will bona fide be the genuinely quantum loop effect.

We will now show how the fermion masses can be generated by means of a self-consistent solution to the Schwinger–Dyson equations. For simplicity we will, in the case of neutrinos, consider just the Dirac masses. Later we shall discuss how the Majorana mass of the right-handed neutrino affects the present mechanism.

II. Simple but crucial is the observation that the assumed fermion mass insertions give rise to generically new contributions to the propagators of the scalar fields. Let us first consider the electrically neutral complex scalars $S^{(0)}$ and $N^{(0)}$. As shown in Fig. 1, the fermion masses induce non-zero two-point functions $\langle S^{(0)} S^{(0)} \rangle$ and $\langle N^{(0)} N^{(0)} \rangle$. We write down explicitly the corresponding proper self-energy (and the following formulas) for the 'southern' scalar,

$$\mu_S^2(p^2) \equiv -i \int \frac{d^4k}{(2\pi)^4} \, \text{Tr}\Big[y_e^\dagger G_e^{LR}(k) y_e^\dagger G_e^{LR}(k+p) \Big] -$$
$$- i \int \frac{d^4k}{(2\pi)^4} \, \text{Tr}\Big[y_d^\dagger G_d^{LR}(k) y_d^\dagger G_d^{LR}(k+p) \Big],$$

where the superscript LR denotes that part of the full propagator G, which in the chiral basis connects the left-and right-handed components of the Dirac field.

Since the scalar mass M_S is expected to change only slightly by the interaction, we may for simplicity evaluate $\mu_S^2(p^2)$ at $p^2 = M_S^2$. We also neglect the one-loop renormalization of the 'ordinary' mass M_S, which can
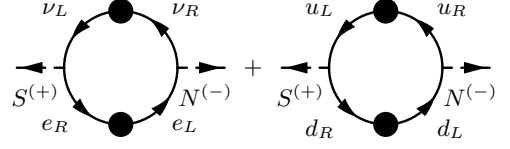
be justified by a suitable choice of the renormalization prescription. The problem thus reduces to finding the spectrum of the bilinear Lagrangian

$$\mathcal{L}_S^{(0)} = \partial_\mu S^{(0)} \partial^\mu S^{(0)\dagger} - M_S^2 S^{(0)} S^{(0)\dagger} -$$
$$- \frac{1}{2}\mu_S^{2*} S^{(0)} S^{(0)} - \frac{1}{2}\mu_S^2 S^{(0)\dagger} S^{(0)\dagger}.$$

Physically observable are then two real spin-0 particles. Their masses are

$$M_{1,2S}^2 = M_S^2 \pm \left| \mu_S^2 \right|.$$

The corresponding real fields S_1 and S_2 are defined through

$$S^{(0)} = \frac{1}{\sqrt{2}} e^{i\alpha_S} (S_1 + iS_2),$$

where the 'mixing angle' α_S is merely given by the phase of the anomalous mass term, $\tan 2\alpha_S = \text{Im}\,\mu_S^2 / \text{Re}\,\mu_S^2$. All formulas written down explicitly for S hold of course also for N.

This is to the mixing of the neutral scalars at the one-loop level. One should, however, note that at higher orders (and, possibly, upon switching on the gauge interaction) we also find transitions between the 'southern' and 'northern' scalars. Consequently, there are altogether four real scalars that mix with one another, and they in fact should because there is no symmetry that would prohibit the mixing.

The case of the charged scalars is considerably simpler. Now only particles with the same charge can mix and they really do as shown in Fig. 2. Analogously to the previous case, we introduce a mixing term

$$-\mu_{SN}^{2*} S^{(+)} N^{(-)} - \mu_{SN}^2 S^{(+)\dagger} N^{(-)\dagger}$$

into the Lagrangian.

The spectrum then contains two charged complex scalars, referred to as $\Phi_1^{(+)}$ and $\Phi_2^{(+)}$, that are mixtures of $S^{(+)}$ and $N^{(-)\dagger}$. Their masses are given by

$$M_{1,2\Phi}^2 = \frac{1}{2}\left[M_S^2 + M_N^2 \pm \sqrt{(M_S^2 - M_N^2)^2 + 4\left|\mu^2\right|^2} \right]$$

and the corresponding field transformation is

$$S^{(+)} = e^{+i\alpha_{SN}} \left(\Phi_1^{(+)} \cos\theta - \Phi_2^{(+)} \sin\theta \right),$$
$$N^{(-)\dagger} = e^{-i\alpha_{SN}} \left(\Phi_1^{(+)} \sin\theta + \Phi_2^{(+)} \cos\theta \right),$$
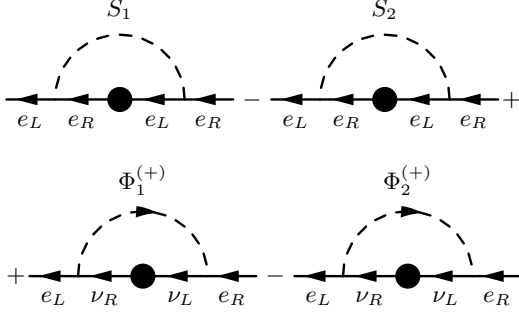
FIG. 3: One-loop contributions to the chirality-changing part of the electron proper self-energy. The minus signs suggest that we encounter the difference of the two scalar propagators. The same graphs also apply to d-quarks, and (upon replacing $S_{1,2}$ with $N_{1,2}$ and reversing the arrows on the charged scalar propagators) neutrinos and u-quarks.

where α_{SN} is the phase of μ_{SN} and

$$\tan 2\theta = \frac{2\left|\mu_{SN}^2\right|}{M_S^2 - M_N^2}.$$

The splittings μ_S^2, μ_N^2 and μ_{SN}^2 of the scalar-particle

masses due to the (yet assumed) dynamical fermion mass generation are both natural and important: First, they come out UV finite due to the large-momentum behavior of $\Sigma(p^2)$ (see below). Second, they manifest spontaneous breakdown of the $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_{em}$ in the scalar sector. Third, they will be responsible for the necessary ultraviolet finiteness of both the fermion and the intermediate vector boson masses.

III. In a self-consistent one-loop approximation the Yukawa interactions (1) give rise to the Schwinger–Dyson equation for the matrix fermion proper self-energy. Since we are interested in symmetry breaking solutions of the Schwinger–Dyson equation, we use the following simplification: We abandon the symmetry-preserving part of the radiative corrections to the fermion propagators that is, we neglect the wave function renormalization [7]. The fermion self-energy thus reduces to its chirality changing parts Σ^{LR} and Σ^{RL}. To simplify the notation, we set $\Sigma \equiv \Sigma^{LR}$.

The Schwinger–Dyson equation then acquires the form, depicted in Fig. 3 explicitly for the case of charged leptons:

$$\Sigma_e(p^2) = \frac{i}{2} \int \frac{d^4k}{(2\pi)^4} \left\{ e^{2i\alpha_S} y_e \Sigma_e^\dagger(k^2) \left[k^2 - \Sigma_e(k^2)\Sigma_e^\dagger(k^2)\right]^{-1} y_e \left[\frac{1}{(p-k)^2 - M_{1S}^2} - \frac{1}{(p-k)^2 - M_{2S}^2} \right] + \right.$$
$$\left. + e^{2i\alpha_{SN}} \sin 2\theta y_\nu \Sigma_\nu^\dagger(k^2) \left[k^2 - \Sigma_\nu(k^2)\Sigma_\nu^\dagger(k^2)\right]^{-1} y_e \left[\frac{1}{(p-k)^2 - M_{1\Phi}^2} - \frac{1}{(p-k)^2 - M_{2\Phi}^2} \right] \right\}. \quad (2)$$

Worth of mentioning is that the assumed fermion mass generation results in the scalar boson mixings which *guarantee* that the kernel of the Schwinger–Dyson equation is Fredholm [8]. In order to proceed we are at the moment forced to resort to simplifications: We neglect the fermion mixing and set $\sin 2\theta = 0$. This amounts formally to neglecting the charged boson mixing. More important, it physically amounts to neglecting an interesting relation between the masses of the up (U) and the down (D) fermions in a weak fermion doublet. With these simplifications, keeping the form of the nonlinearity unchanged, we perform in Eq. (2) the Wick rotation, angular integrations, and Taylor expand in $M_{1S}^2 - M_{2S}^2$. For a generic (say e) fermion self-energy in dimensionless variables $\tau = p^2/M^2$ we obtain

$$\Sigma(\tau) = \beta \int_0^\infty d\sigma \frac{\sigma \Sigma(\sigma)}{\sigma + \frac{1}{M^2}\Sigma^2(\sigma)} \kappa(\sigma, \tau), \quad (3)$$

where

$$\kappa(\sigma, \tau) \equiv \frac{[(\sigma + \tau + 1)^2 - 4\sigma\tau]^{-1/2}}{\sigma + \tau + 1 + [(\sigma + \tau + 1)^2 - 4\sigma\tau]^{1/2}}$$

and $\beta = (y^2/8\pi^2)(M_1^2 - M_2^2)/M^2$. Here M is the common mean mass about which M_1 and M_2 are expanded. Numerical analysis of the Eq. (3) reveals the existence of a solution $\Sigma(\tau)$, albeit yet for large values of β [8]. It has a form similar to the step function rapidly approaching zero after the step at $\tau = 1$. It correctly exhibits the low-momentum origin of the fermion masses. The model can pretend to phenomenological relevance only after demonstrating strong amplification of fermion masses as a response to small changes of preferably small Yukawa couplings. This work is in progress.

For the electrically charged fermions $\Sigma(p^2)$ defines the fermion mass m as the solution of the equation $m = \Sigma(p^2 = m^2)$. The case of neutrinos is more subtle and requires further work: (i) Without introducing ν_R neutrinos would be massless in the present model. (ii) With ν_R
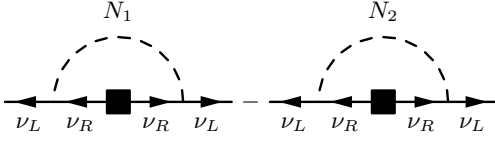
FIG. 4: One-loop contributions to the Majorana mass of the left-handed neutrinos. The solid square denotes the right-handed Majorana mass.
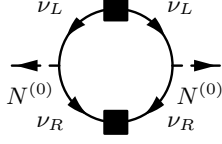

FIG. 5: One-loop contribution to the $N^{(0)} - N^{(0)\dagger}$ mixing arising from the neutrino Majorana mass term.

the mechanism described above generates the UV-finite Dirac neutrino self-energy Σ_ν. Moreover, there is also a 'hard' mass term present in $\mathcal{L}$,

$$\overline{(\nu_R)^C} M_M \nu_R + \text{H.c.} \tag{4}$$

Finally, due to (4) the model is in general capable of generating a UV-finite left-handed Majorana mass matrix, the mechanism being essentially the same as that for the Dirac masses, see Fig. 4. Strictly speaking, it is more appropriate to treat the Majorana masses self-consistently, on the same footing as the Dirac masses. The mixing amplitude of the neutral 'northern' scalars then receives an additional contribution shown in Fig. 5.

Consequently, with ν_R the model should describe $2n_f$ massive Majorana neutrinos with a generic see-saw spectrum.

IV. Dynamically generated fermion proper self-energies $\Sigma(p^2)$ break spontaneously the $SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_{em}$. Consequently, the W and Z bosons dynamically acquire masses. To determine their values we have to calculate residues at single massless poles of their polarization tensors [9].

(i) The massless poles are those of the 'would-be' Nambu–Goldstone (NG) bosons. They are visualized in the proper vertex functions Γ_W^α and Γ_Z^α as necessary consequences of the Ward–Takahashi identities [10]:

$$\Gamma_W^\alpha(p+q,p) \xrightarrow{q^2 \to 0} \frac{g}{2\sqrt{2}}\{\gamma^\alpha(1-\gamma_5)-$$
$$-\frac{q^\alpha}{q^2}[(1-\gamma_5)\Sigma_U(p+q) - (1+\gamma_5)\Sigma_D(p)]\},$$

$$\Gamma_Z^\alpha(p+q,p) \xrightarrow{q^2 \to 0} \frac{g}{2\cos\theta_W}\{t_3\gamma^\alpha(1-\gamma_5)-$$
$$-2Q\gamma^\alpha \sin^2\theta_W + \frac{q^\alpha}{q^2}t_3[\Sigma(p+q)+\Sigma(p)]\gamma_5\}.$$

(ii) From the pole terms in Γ_W^α and Γ_Z^α we extract the effective vertices between the gauge and three multi-component 'would-be' Nambu–Goldstone bosons. They are given in terms of the UV-finite tadpole loop integrals

$$J_W^\alpha(q) = \text{Tr}\int \frac{d^4k}{(2\pi)^4} P_- G_U(k+q)\frac{g}{2\sqrt{2}}\gamma^\alpha(1-\gamma_5)G_D(k) = \frac{g}{\sqrt{2}}\frac{1}{N_W}[I_{U;D}^\alpha(q) + I_{D;U}^\alpha(q)],$$

$$J_Z^\alpha(q) = \text{Tr}\int \frac{d^4k}{(2\pi)^4} P_0 G(k+q)\frac{g}{2\cos\theta_W}\left[t_3\gamma^\alpha(1-\gamma_5) - 2Q\gamma^\alpha\sin^2\theta_W\right]G(k) = \frac{g}{2\cos\theta_W}\frac{1}{N_Z}[I_{U;U}^\alpha(q)+I_{D;D}^\alpha(q)],$$

where

$$I_{U;D}^\alpha(q) \equiv 4n_c\int\frac{d^4k}{(2\pi)^4}\frac{\Sigma_U^2(k+q)k^\alpha}{[(k+q)^2 - \Sigma_U^2(k+q)][k^2 - \Sigma_D^2(k)]} \equiv -iq^\alpha I_{U;D}(q^2).$$

Also $G(k) = [\slashed{k} + \Sigma(k)]/[k^2 - \Sigma^2(k)]$, $P_\pm = N_W^{-1}[(1 \mp \gamma_5)\Sigma_U(p+q) - (1\pm\gamma_5)\Sigma_D(p)]$, $P_0 = N_Z^{-1}\gamma_5 t_3[\Sigma(p+q) + \Sigma(p)]$, and $n_c = 1$ for leptons and $n_c = 3$ for quarks. The normalization factors N_W, N_Z are defined by the mass sum rules below. In the loop integrals summation over all families of U and D fermions (both leptons and quarks) is implied.

(iii) The effective gauge-boson–'would-be' NG vertices

immediately give rise to the longitudinal parts of W and Z polarization tensors with massless 'would-be' NG poles. Their residues are

$$m_W^2 = \frac{1}{4}g^2\sum(I_{U;D}(0) + I_{D;U}(0)) \equiv \frac{1}{4}g^2 N_W^2,$$

$$m_Z^2 = \frac{1}{4}(g^2+g'^2)\sum(I_{U;U}(0)+I_{D;D}(0)) \equiv \frac{1}{4}(g^2+g'^2)N_Z^2.$$

If the proper self-energies Σ_U and Σ_D were degen-

erate, not surprisingly the Standard model relation $m_W^2/m_Z^2 \cos^2\theta_W$ would be fulfilled. Quantitative analysis of departure from this relation demands quantitative knowledge of the functional form of the proper self-energies. At present we can only refer to an illustrative model analysis of Ref. [11].

V. Generating the lepton, quark, and vector boson masses spontaneously is a theoretical necessity. Principles are, however, more general then their known realizations. Being genuinely quantum and non-perturbative the mechanism of spontaneous mass generation suggested here is rather stiff:

(i) It relates all quark (lepton) dynamically generated proper self-energies $\Sigma(p^2)$ with each other. After some labor the relations between fermion self-energies should turn into relations between the quark (lepton) masses, their corresponding mixing angles and the CP-violating phase(s). (ii) It relates all dynamically generated proper self-energies $\Sigma(p^2)$ to m_W and m_Z. (iii) There is no generic weak-interaction mass scale $v \simeq 246\,\mathrm{GeV}$ in the present model. The mass scale of the world is fixed by the masses M_N, M_S and M_M.

We believe that with new experimental data soon to come the model might provide a slit into yet unknown short-distance particle dynamics. Be it as it may it predicts four electrically neutral scalar bosons and two charged ones. They should be heavy, but not too much.

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